Instantaneous Concentration Fluctuations in Point-Source Plumes

A model for the variance σ_c^2 of instantaneous plume concentrations, at a level corresponding to that of the Gaussian formulas for the mean field, is developed and tested against available data. Starting from the Eulerian species transport equation, theoretical and empirical information is used to estimate parameters and simplify the form of the transport equation for σ_c^2 in a meandering frame of reference. Then, introduction of a localized production of fluctuations (LPF) scheme allows construction of analytical expressions for σ_c^2 that provide a direct means for calculating the concentration variance inside the instantaneous boundaries of plumes resulting from continuous point emissions in turbulent flows with uniform mean velocity.

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Introduction

Prediction of expected concentration fluctuation levels in point-source plumes is a key need that arises in many problems related to turbulent dispersion. Typical examples are:

1. Estimation of quantitative measures for the inherent uncertainty in models of contaminant dispersion in the environment. This uncertainty is associated with the stochastic nature of the dispersion phenomenon per se, as opposed to the potentially reducible uncertainty associated with errors and approximations in the model structure and the input data (Fox, 1984; Weil, 1985).
2. Modeling situations concerned with the exceeding of some critical value by a rapidly changing concentration, even for very short times. Examples of such situations are the accidental release of toxic or flammable gases (Chatwin, 1982), and the creation of smoke screens for defense purposes (Ohmstede and Cogan, 1982). In these cases probabilistic properties of the concentration field are essential in assessing the environmental impact.
3. Modeling nonlinear processes (usually chemical) within plumes. For processes such as reactions with nonlinear kinetics, the effective conversion rates may depend critically on the level and spatial distribution of turbulent concentration fluctuations (i.e., on the quality or completeness of the fine-scale mixing locally inside the instantaneous plume boundaries). The local intensity of segregation I_s, involving the variance of fine-scale "in-plume" fluctuations, can be used to quantify the interaction of *mixing and chemistry* for second-order chemical reactions.

In dealing with such problems it is essential to discern the spatial scales associated with a given portion of the fluctuations spectrum, as they may affect the phenomenon under study and its consequences in qualitatively very different ways. Thus turbulent eddies that at a given location are of size comparable to and larger than the local plume dimensions result in its irregular meandering, i.e., a bulk motion (Figure 1). Only eddies smaller than these are responsible for the mixing process inside the instantaneous plume boundaries, the state of which is described by the level of instantaneous "internal" concentration fluctuations and thus is associated with the processes of relative or two-particle dispersion. Thus, for example, rates of nonlinear chemical reactions between plume constituents and the ambient are affected solely by the internal fine-scale fluctuations. On the other hand, assessing physiological effects from the varying concentration of a pollutant requires taking into account the total variability of the concentration field at the fixed receptor location.

A complete description of the fluctuations problem would require knowledge of the entire probability density of random concentrations, and in the case of interacting concentration fields (e.g., of chemically reacting species) of their joint probability densities. Probability densities at every point in a fixed frame of reference, and the associated statistics, reflect the total randomness of the concentration field that results from absolute diffusion; the respective densities and statistics for every point in a frame of reference whose origin follows the random meandering motion of the center of mass of an instantaneous release, or the instantaneous centerline of a continuous plume, reflect internal randomness due to relative dispersion. To deduce fixed-frame probability densities and noncentral moments of concentrations from the corresponding moving-frame quantities one

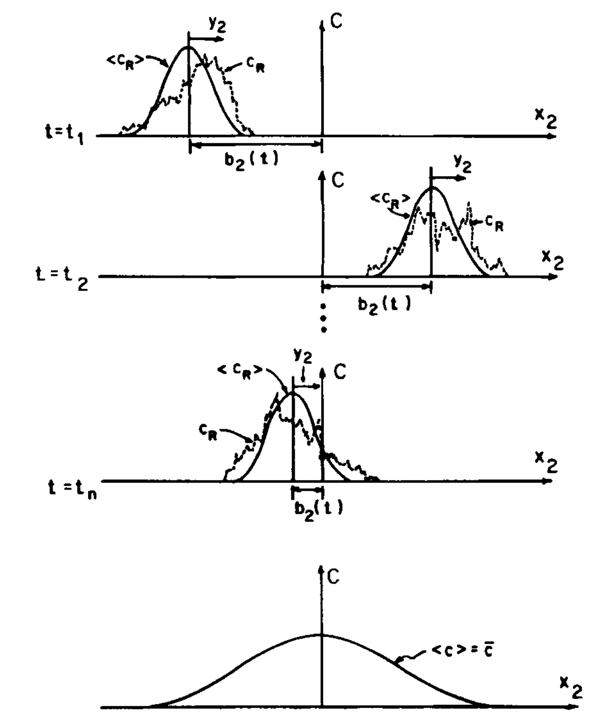

Figure 1. Crossflow profiles of expected instantaneous concentrations in fixed $\langle c\rangle$ and meandering $\langle c_R\rangle$ frames of reference.

Downwind distance x_1 is the same at various times for a plume assumed to be transferred essentially intact by meandering. Instantaneous realizations of concentration field $c \equiv c_R$ are also presented. $\langle c\rangle$ is the long-term time average of the ensemble average $\langle c_R\rangle$ at a fixed position.

has to calculate the convolution of the latter with the spatial position probability density of the meandering origin of the moving frame (Csanady, 1973, Chs. 4, 7).

Although the problem of calculating probability densities of concentration fields (of both conserved and reactive scalars) has been pursued through a variety of approaches (Hill, 1976; O'Brien, 1980; Pope, 1982, 1985), its complexity does not presently allow for simple, practical models (Georgopoulos and Seinfeld, 1986, Chs. 4, 5). A more feasible goal is the prediction of the second moment $\langle c^2\rangle$, or of the variance σ_c^2, of the random instantaneous concentration field, which, combined with the knowledge of the mean $\langle c\rangle$, would provide a description adequate for most applications. Theoretical study of the σ_c^2 dynamics originated in the works of Corrsin (1952, 1964) and Batchelor (1959). Major results concerning σ_c^2 behavior in different turbulent flows are summarized in various sources (Brodkey, 1967; Monin and Yaglom, 1971, 1975; Hinze, 1975; Townsend, 1976; Bradshaw, 1978; Fischer et al., 1979). As far as dispersion in ambient turbulence is concerned, the three major approaches commonly employed in modeling the mean field—i.e., Eulerian and Lagrangian statistical methods (including Langevin models) and dimensional (similarity) analysis—have also been used, often in combination, to study the variance of concentration fields resulting from passive releases from strongly localized sources (Csanady, 1967, 1973; Thomas, 1979; Chatwin and Sullivan, 1979a, 1980; Robins and Fackrell, 1979; Durbin, 1980, 1982; Sawford, 1982, 1983, 1985; Sykes et al., 1984; Hanna, 1984; see also Weil, 1985). (The term "passive" is used in the sense that this release does not affect the properties of the ambient flow.) A separate class of models originated with Gifford's (1959) fluctuating plume concept, which considers fluctuations produced exclusively by the bulk meandering of the plume ("external" fluctuations), neglecting all randomness inside the instantaneous plume boundaries, and therefore calculates what we will call here the external variance. Various applications and extensions of this concept (Scriven, 1965; Diamante et al., 1976; Fackrell and Robins, 1982b) as well as related formulations (Venkatram, 1979; Hanna, 1984) have appeared in the literature. Finally, the empirical models of Wilson et al. (1982a, b) provide expressions for σ_c^2 constructed so as to fit wind tunnel data where meandering was recognized as the dominant source of observed fluctuations (Fackrell and Robins, 1982a, b).

Available data of short-term fluctuation statistics for pure plumes from point sources, that is, for dispersion governed exclusively by the ambient turbulence, are basically relevant to the total variance observed at a fixed point, and include mainly wind tunnel measurements (Fackrell, 1978, 1980; Fackrell and Robins, 1981, 1982a, b; Robins, 1978, 1979; Gad-el-Hak and Morton, 1979) and atmospheric field measurements (Gosline, 1952; Barry, 1971; Ramsdell and Hinds, 1971; Kimura et al., 1981; Jones, 1983; Sawford et al., 1985). Data on in-plume fluctuations, definitely more scarce, are also available, both from laboratory flows with insignificant meandering (Becker et al., 1966), and from field measurements performed relevant to the meandering center of mass of continuous oceanic plumes (Murthy and Csanady, 1971; Sullivan, 1971; Chatwin and Sullivan, 1979b) and atmospheric plumes (Eidsvik, 1980). Finally, some related information can be found in the substantial fluid-mechanical literature on momentum jets and buoyant plumes (List, 1982; Gebhard et al., 1984).

Some rather general results on plume fluctuations, based mainly on data from pipe flows and from wind tunnels simulating either homogeneous and isotropic turbulence or the neutral atmospheric boundary layer are:

1. Production of both internal and external fluctuations is in general significant only close to the source.

2. Meandering is typically the most significant source of fluctuations in the near field, whereas internal fluctuations prevail far downwind. Further analysis suggests that the external intensity of fluctuations at the centerline (i.e., the ratio of external variance to the square of the mean concentration) reaches a maximum at some distance downwind and decays toward zero thereafter; the corresponding internal intensity does not decay but seems to tend toward some constant nonzero value.

3. Intermittency effects are very significant in the near field and are typically associated with meandering; relative concentration measurements are very often free of intermittency effects in the core of the instantaneous plume.

4. The variance of atmospheric concentrations from ground-level sources exhibits profiles that are approximately self-similar in both the horizontal and vertical directions; further, it does not show significant dependence on source size.

5. The same variance for elevated sources initially shows dependence on source size that is eventually "forgotten." Horizontal profiles of σ_c^2 are again approximately self-similar, but vertical profiles show a more complicated behavior: In the immediate vicinity of the source they are self-similar until the effect of the ground is felt. In the far field, however, these pro-

files become again self-similar, resembling those of a ground-level source.

6. A power law concentration probability density resulting from Gifford's fluctuating plume model seems to provide the best fit to experimentally measured densities of fixed-frame data in most cases. Log-normal densities offer the best fit to sets of nonintermittent data.

However, in spite of the recent advances in analyzing and understanding the problem of turbulent concentration fluctuations, a simple, rational scheme for routine calculation of the instantaneous internal plume concentration variance, for use in conjunction with the Gaussian relative dispersion formulas for the instantaneous mean field does not exist. Such a model can, in fact, be viewed as a counterpart of Gifford's (1959) model for the external variance. The development of such a practical scheme is the object of this work. We start from the Eulerian transport equation for σ_c^2, modeling the process described by its components in terms of known or measurable quantities, and continuing with an analysis of potential simplifications of the mathematical description through self-similarity assumptions for σ_c^2. The information that is systematized in this way is subsequently utilized in the formulation of a new model that provides simple, closed form, analytical expressions for σ_c^2 for the case of a continuous passive "point" release of material in a turbulent field of uniform mean velocity.

Transport Equation for Concentration Variance

General considerations

Eulerian models for the estimation of σ_c^2 are based on the fundamental transport equation for c for a fixed frame of reference

$$\frac{\partial c}{\partial t} + u_i \frac{\partial c}{\partial x_i} = D \frac{\partial^2 c}{\partial x_i \partial x_i} \quad (1)$$

(summation convention implied) where u_i, c are stochastic variables that can be viewed as consisting of a mean and a fluctuating part (Reynolds decomposition), i.e., $u_i = \langle u_i \rangle + u_i'$, $c = \langle c \rangle + c'$. In the following the operation $\langle \cdot \rangle$ always denotes ensemble averaging; for locally homogeneous and stationary turbulence this can be replaced by spatial or temporal averaging, under an ergodic hypothesis (Hinze, 1975) as far as the velocities are concerned. For strongly localized sources the concentration field cannot be homogeneous and thus only time averages can approximate ensemble means (when, of course, the specific phenomenon under study is in steady state).

The equation for the variance of c as obtained from Eq. 1 is

$$\frac{\partial \langle c'^2 \rangle}{\partial t} + \overbrace{\langle u_i \rangle \frac{\partial \langle c'^2 \rangle}{\partial x_i}}^{\text{(i)}} = \overbrace{-2\langle u_i c' \rangle \frac{\partial \langle c \rangle}{\partial x_i}}^{\text{(ii)}} + \overbrace{\frac{\partial}{\partial x_i}\left(D \frac{\partial \langle c'^2 \rangle}{\partial x_i} - \langle u_i' c'^2 \rangle\right)}^{\text{(iii)}} \overbrace{-2D \frac{\langle \partial c' \partial c' \rangle}{\partial x_i \partial x_i}}^{\text{(iv)}} \quad (2)$$

This equation expresses the fact that the level of $\sigma_c^2 \equiv \langle c'^2 \rangle$ changes through an imbalance of

(i) advection

(ii) the generation rate of scalar fluctuations by gradients in the mean concentration

(iii) the diffusive transfer produced by molecular dispersion and turbulent velocity fluctuations (the former being usually negligible)

(iv) the dissipation of fluctuations due to molecular diffusion in the fine-scale structure.

The relative importance of the different processes in the σ_c^2 budget depends on the particular type of flow (Launder, 1978). Bulk meandering and internal fine-scale motions will contribute in a qualitatively different manner not only to the observed overall level of fluctuations at a given point but to the relative balance of terms in the governing equation for σ_c^2 as well. Here we confine attention to relative dispersion in turbulence with a uniform mean velocity $\bar{u}$, and to internal fluctuations in plumes that generate an ensemble of instantaneous realizations which is in a steady state with respect to a frame of reference that follows the randomly meandering centerline translating parallel to itself. (The steady state concept here is, of course, relevant to the mean of the ensemble and not to the actual concentration field.) Equations 1 and 2 with $\partial(\cdot)/\partial t = 0$, $\langle u_1 \rangle = \bar{u}$, $\langle u_2 \rangle = \langle u_3 \rangle = 0$ are assumed to hold for this moving frame of reference. In other words, if the moving frame coordinates in the crosswind plane at x_1 are y_2, y_3 where $y_2 = x_2 - b_2$, $y_3 = x_3 - b_3$, b_2, b_3 being the random coordinates of the instantaneous plume centerline in this plane, then Eqs. 1–2 are assumed to adequately describe mass transport in the meandering frame (see Georgopoulos and Seinfeld, 1986, Ch. 6, for a detailed discussion of this point). In the case where flow conditions are such that the mean plume centerline is not a straight line parallel to the horizontal plane (i.e., $\langle b_k \rangle$, $k = 2, 3$, are not constant for all x_1), then the above equations are still sufficient approximations (for a translating frame meandering about this centerline) for very small values of the derivatives $(\partial \langle b_k \rangle / \partial x_l)$, $k = 2, 3$, $l = 1, 2, 3$. The situation considered here is schematically represented in Figure 1: The mean concentration in Eq. 1 is $\langle c_R \rangle$ and the fluctuations in Eq. 2 represent deviations of actual realizations c_R from this value. In the present work we limit attention to c_R and $\langle c_R \rangle$; fixed frame properties (such as $\langle c \rangle$ of Figure 1) will not be examined. Thus in the following the subscript R for the concentration will be dropped without any loss of clarity. Another point to note is that in this approach intermittency effects are attributed to bulk motions, in compliance with available experimental evidence; the probability of exactly zero concentrations in the vicinity of the origin of the meandering frame is assumed negligible.

Modeling individual terms of the variance transport equation

Production Term. In the perspective of this study we will consider as adequate a description of $\langle u_i' c' \rangle$ in terms of eddy diffusivities K_{R_i} that will be assumed to be in general functions of the distance from the source, and to correspond to the effects of small-scale dispersion processes (the subscript R_i is used to denote the relevance to relative dispersion, or absolute dispersion without significant meandering). In this way Eq. 2 continues to hold locally inside the instantaneous plume. In a sense this is a Lagrangian modeling step, since K_{R_i}'s thus defined are not properties of the flow field but functions of the dispersion time for specific emissions. Thus, locally

$$\langle u_i'c' \rangle = -K_{R_i}(x_1)\frac{\partial \langle c \rangle}{\partial y_i} \tag{3}$$

where the point species source is located at $x_1 = 0$. The variation of K_{R_i} with downwind distance from the source will be calculated from

$$K_{R_i}(x_1) = \frac{\langle u_1 \rangle}{2}\frac{d\sigma_{Ri}^2}{dx_1} = \frac{1}{2}\frac{d\sigma_{Ri}^2}{dt} \tag{4}$$

where σ_{R_i} is the standard deviation of relative dispersion in the i direction. Methods for estimating σ_R's can be found in Hinze (1975, p. 406), Monin and Yaglom (1975), and Seinfeld (1983). Thus

$$\Pi_c = -2\langle u_i'c' \rangle \frac{\partial \langle c \rangle}{\partial y_i} = 2K_{R_i}(x_1)\left(\frac{\partial \langle c \rangle}{\partial y_i}\right)^2 \tag{5}$$

Diffusive Flux Term. Most approaches for modeling the diffusive flux of σ_c^2 have also adopted a gradient representation of $\langle u_i'c'^2 \rangle$, usually neglecting all molecular diffusion effects (Launder, 1978). Various forms of gradient type formulas have been used (Bradshaw and Ferris, 1968; Spalding, 1971; Wyngaard, 1975; Thomas, 1979; Sykes et al., 1984). A simple approach, especially when eddy diffusivities are used in representing $\langle u_i'c' \rangle$, is to assume a gradient transport relationship of the form

$$\langle u_i'c'^2 \rangle - D\frac{\partial \sigma_c^2}{\partial x_i} = -\tilde{K}_i\frac{\partial \sigma_c^2}{\partial x_i} \tag{6}$$

Assuming that the same dispersive mechanisms account for the spread of both $\langle c \rangle$ and σ_c^2, we set $\tilde{K}_i = K_{R_i}$. Data from geophysical flows provide supportive but certainly not conclusive indication for the validity and the limitations of such a gradient transport scheme (Csanady, 1973; Netterville and Wilson, 1980). In any case, since higher order closure schemes are beyond the scope of the present analysis, we will adopt the closure assumption of Eq. 6 with $\tilde{K}_i = K_{R_i}$ given by Eq. 4.

Dissipation Term. Many studies have attempted to model this term by analogy to the dissipation of velocity fluctuations (kinetic energy dissipation), for which there is more extensive experimental information available. The most common procedure is to adopt an expression of the general form

$$\Phi = 2D\frac{\langle \partial c' \partial c' \rangle}{\partial y_i \partial y_i} = \frac{nD\sigma_c^2}{\ell_d^2} = \frac{\sigma_c^2}{t_d} \tag{7}$$

where ℓ_d is a dissipative length scale (a hybrid Corrsin scale) analogous to the Taylor scale for the dissipation of velocity fluctuations, and $nD/\ell_d^2 = 1/t_d$ is the reciprocal of a characteristic decay time scale t_d. The choice of the numerical factor n in this relation is a matter of convention (e.g., $n = 4$, 6, and 12 are used in the literature). The time scale t_d is the single most important quantity in the characterization of the mixing process; actually, in most approaches all the effects of molecular diffusion on mixing are lumped into this parameter.

For the case of homogeneous, quasi-isotropic, turbulent velocity and concentration fields, both theoretical considerations and experimental evidence (Gibson and Schwarz, 1963; Hinze, 1975; Launder, 1978; Warhaft and Lumley, 1978; Sreenivasan et al., 1980; Durbin, 1982) suggest that

$$t_d = kt + k_0 \tag{8}$$

where k lies in the range 1/3 to 2/3, $t = x_1/\langle u_1 \rangle$, and k_0 is a constant that can be assumed equal to zero when the production of fluctuations is localized at $t = 0$.

In the case of a continuous plume generated by a concentrated point (or line) source in a field of homogeneous turbulence we may also expect the rate of dissipation of concentration fluctuations to be proportional to fluctuation intensity σ_c^2, because essentially the same physical factors must govern across-the-spectrum transfer of contributions to σ_c^2, regardless of the manner in which the fluctuations were generated (Csanady, 1973). However, now the "ages" of the concentration fluctuations cover a broad range and the decay time scale may vary in an unknown manner. Thus one should set locally $\Phi = \sigma_c^2/t_d$ with $t_d = t_d(x_1, y_2, y_3)$, i.e., assume that t_d is some function of position that has to be determined.

The approach described by Eq. 7 has appeared in some works relevant to air pollution. Thus Donaldson and Hilst (1972) estimated a typical (constant) value of $t_d \simeq 5$ min for a (hypothetical) average turbulent mass of atmospheric air. This (constant) value of the decay time scale was used by Kewley (1978) in a reactive plume model. However, in plumes the factors affecting the intensity of dissipation (and therefore t_d) will change significantly with travel time, and the assumption of constant t_d is not an appropriate one. In a more justifiable approach Csanady (1967, 1973) and Thomas (1979) adopted Eq. 8 with the theoretical value $k = 2/3$ (Hinze, 1975, p. 301) and $k_0 = 0$. Modified forms of Eq. 8 were also suggested by Fackrell and Robins (1979) and Netterville (1979) and utilized by Wilson et al. (1982a, b) in an empirical model for the total level of atmospheric plume fluctuations. However in the latter case the dominant component in the overall observed variance values was bulk variance (Fackrell and Robins, 1982a, b), the dissipation of which mainly reflects the expansion of the instantaneous plume to the size of the time-average envelope; the approach of Sykes et al. (1984) is more appropriate for this situation.

For the dissipation of fine-scale fluctuations by molecular diffusion in the moving frame of reference we adopt Eq. 8 in the form

$$t_d = \frac{x_1 + x_0}{A_1\langle u_1 \rangle} = \frac{1}{A_1}(t + t_0) \tag{9}$$

where $A_1 = 1/k$ and x_0 is a "virtual origin correction" that accounts for the initial production-dominated region near the source. This equation should be viewed as a reasonable *first estimate* for $t_d(x_1, y_2, y_3)$ for a relatively slender plume. The success of this approximation for a given range of downwind distances will rely heavily on the proper choice of A_1; unfortunately, the uncertainty involved in this choice is large, even for relatively ideal flow situations. Some further insight on this problem can be obtained by examining the transport education for Φ (Launder, 1978). Indeed, for point sources the generation terms involving mean concentration field gradients will play a significant role in the overall Φ balance, especially in the vicinity of the source, thus resulting in higher dissipation rates and lower characteristic dissipation times in comparison with the quasi-isotropic cases to which most of the available information is relevant.

Effects of boundaries

The presence of a boundary parallel to the mean flow $\bar{u}$ (e.g., the ground in the case of atmospheric dispersion) affects the balance of σ_c^2 in two ways.

First, if this boundary does not interact chemically or otherwise with the plume species, it imposes a condition of zero transfer of plume material, which, in addition to increasing the mean concentration near the surface, affects the intensity of concentration fluctuations by controlling the production of σ_c^2. Since $\partial\langle c\rangle/\partial x_1$ is normally small compared to the lateral gradients, a decrease in $\partial\langle c\rangle/\partial y_3$ will reduce the production of fluctuations significantly, especially near the horizontal centerline of the plume where $\partial\langle c\rangle/\partial y_2$ will also be small.

Second, the boundary affects the flow field in such a way that advection and turbulent transfer terms are expected to be small near the surface. In this analysis it will be assumed that the mean velocity is uniform except for a very thin layer near the boundary. However the no-slip boundary condition near the surface results in high local mean shear and intensity of turbulence which rapidly distort and stretch plume filaments, thus increasing the surface area available for molecular diffusion, which dissipates concentration fluctuations. Thus, in general the presence of production and dissipation processes accounts for different behavior of $\langle c\rangle$ and σ_c^2 near the surface. Wind tunnel studies suggest that very close to the ground there might be a well-mixed layer, where dissipation practically reduces σ_c^2 to zero; however, available data do not extend close enough to the surface to show explicitly this effect (Wilson et al., 1982a).

Hence, $\partial\sigma_c^2/\partial y_3$ is not expected to approach zero gradually at the surface. It is more appropriate to view the latter as an absorbing (possibly not perfectly) boundary with respect to σ_c^2 and thus

$$\sigma_c^2 \rightarrow 0 \quad \text{at } x_3 = y_3 + b_3 = 0 \tag{10}$$

Effects of source size

The assumption of a point source is an extreme idealization that is actually incompatible with the process of relative diffusion, since the latter requires a nonzero initial separation of the diffusing fluid particles (Durbin, 1980). The degree to which concentration fluctuations are influenced by source conditions, such as source size (or initial separation) has been a subject of both theoretical analysis (Chatwin and Sullivan, 1979a; Durbin, 1980, 1982; Sawford, 1983), and experimental study (Fackrell and Robins, 1982a). The available experimental evidence for continuous plumes relates important source effects to meandering processes and shows that they persist for distances where bulk fluctuations are dominant; far downstream the variance tends to "forget" these effects. Theoretical considerations (Durbin, 1980; Sawford, 1983) show that the intensity of internal fine-scale fluctuations tends to a constant value that in general must depend on the initial size of an instantaneous release. However, the available data on relative dispersion of continuous plumes are not adequate to provide reliable quantitative estimates of source size effects. In fact, far enough from the source, data on both the total and the fine-scale variance, and for both elevated and ground level sources, show that under constant flow conditions the centerline intensity of fluctuations approaches a constant value that is (almost) independent of source size. In the present work, in order to retain simplicity, source effects will not be accounted for explicitly; the species source is assumed localized at a point and necessary corrections to this idealization are invoked *a posteriori* when the mathematical manipulations cannot accommodate the point-source concept. The effects of the finite size of the actual source will have to be incorporated (either explicitly or implicitly) in a parameter of the model.

Assumption of self-similarity

Introducing the approximation of Eq. 9 and the transport closure schemes of Eqs. 3 and 6, Eq. 2 reduces to the following form for the steady state [in the (x_1, y_2, y_3) frame] point-source plume in a mean flow field $\bar{u} = \langle u_1\rangle$ along the $x_1 \equiv y_1$ direction:

$$\overbrace{\bar{u}\frac{\partial\sigma_c^2}{\partial x_1}}^{\text{(i)}} = \overbrace{2K_{R_2}(x_1)\left[\left(\frac{\partial\langle c\rangle}{\partial x_1}\right)^2 + \left(\frac{\partial\langle c\rangle}{\partial y_2}\right)^2\right] + 2K_{R_3}(x_1)\left(\frac{\partial\langle c\rangle}{\partial y_3}\right)^2}^{\text{(ii)}} + \overbrace{K_{R_2}(x_1)\frac{\partial^2\sigma_c^2}{\partial y_2^2} + K_{R_3}(x_1)\frac{\partial^2\sigma_c^2}{\partial y_3^2}}^{\text{(iii)}} - \overbrace{\frac{\sigma_c^2}{t_d(x_1)}}^{\text{(iv)}} \tag{11}$$

where terms (i) to (iv) represent the respective terms of Eq. 2.

A rational approach toward the simplification of Eq. 11 is based on the hypothesis of self-similarity for both the instantaneous $\langle c\rangle$ and σ_c^2 fields. As already mentioned, this hypothesis has considerable experimental support (Csanady, 1973; Fackrell and Robins, 1982a); it was first introduced as an approximation in the Eulerian modeling of σ_c^2 by Csanady (1967) who studied the construction of self-similarity models for isotropic turbulence and for an unbounded flow.

If Eq. 11 is satisfied by $\langle c\rangle$ and σ_c^2 that obey the self-similar relations

$$\langle c\rangle = \langle c_0(x_1)\rangle f(\hat{r}), \quad \sigma_c^2 = \langle c_0(x_1)\rangle^2 g(\hat{r}) \tag{12}$$

where

$$\langle c_0(x_1)\rangle = \frac{S}{2\pi\bar{u}\sigma_{R_2}\sigma_{R_3}}, \quad f(\hat{r}) = \exp\left(-\frac{\hat{r}^2}{2}\right)$$

(S is the source strength) and $\hat{r} = \check{r}/\check{s}$, with $\check{r} = \sqrt{y_2^2\sigma_{R_3}^2 + y_3^2\sigma_{R_2}^2}$, $\check{s} = \sigma_{R_2}\sigma_{R_3}$ [notice that $\check{r}$, $\check{s}$ have units of (length)^2], then it can be shown that two necessary conditions for Eq. 11 to have self-similar solutions are

$$\frac{d\sigma_{R_3}}{d\sigma_{R_2}} = \frac{\sigma_{R_3}}{\sigma_{R_2}} \quad \text{or } \sigma_{R_3} = \kappa\sigma_{R_2} \tag{13}$$

and

$$\frac{\sigma_{R_2}\sigma_{R_3}}{\bar{u}t_d} = \hat{\alpha}\left(\frac{\sigma_{R_2}^3 y_3^2}{\check{r}^2}\frac{d\sigma_{R_3}}{dx_1} + \frac{\sigma_{R_3}^3 y_2^2}{\check{r}^2}\frac{d\sigma_{R_2}}{dx_1}\right) \tag{14}$$

where κ and $\hat{\alpha}$ are constants.

Here we will in general assume that the increase of σ_{R_2} and σ_{R_3} with distance from the source obeys locally the same exponential law within a multiplicative factor. (This exponential law will be different in the various phases of relative dispersion.) Regarding atmospheric dispersion, experience shows the above assumption to be usually a reasonable approximation.

Now, for $\sigma_{R_3} = \kappa\sigma_{R_2}$ one has

$$\hat{\alpha} = \frac{\sigma_{R_2}}{\left(\dfrac{d\sigma_{R_2}}{dx_1}\right)} \frac{1}{\bar{u}t_d}$$

If σ_{R_2} obeys the power law $\sigma_{R_2} = \sigma_{0_2}x_1^p$, σ_{0_2} being a constant of appropriate dimensions, then $\hat{\alpha} = (x_1/p)(1/\bar{u}t_d)$, and, employing Eq. 9,

$$\hat{\alpha} = \frac{A_1}{p}\frac{x_1}{x_1 + x_0}$$

Thus, if A_1 and p are constants over a finite range of x_1, the necessary condition for self-similar solutions of Eq. 11 becomes $x_0 = 0$, in which case $\hat{\alpha} = A_1/p$. Hence, the theoretical and empirical information that is available for p and A_1 can be used to provide first estimates for $\hat{\alpha}$.

When Eqs. 13 and 14 hold, Eq. 11 becomes

$$\frac{d^2g}{d\hat{r}^2} + \left(\frac{1}{\hat{r}} + \hat{r}\right)\frac{dg}{d\hat{r}} + (4 - \hat{\alpha})g = -2\left(\frac{df}{d\hat{r}}\right)^2 \quad (15)$$

The boundary conditions for Eq. 15 arise from requirements of axial symmetry and a decay of σ_c^2 to zero at large radial distances:

$$\frac{dg}{d\hat{r}} = 0 \quad \text{at } \hat{r} = 0, \quad g \rightarrow 0 \text{ at } \hat{r} \rightarrow \infty \quad (16)$$

We must remark here that boundary effects, which would complicate not only the formulation of boundary conditions but also the appropriate choice for $\hat{r}$, are not considered in the above analysis. Therefore this approach is formally valid for unbounded domains. Furthermore, for an elevated source the existence of ground effects imposes an "external" length scale on the dispersion process. This destroys the conditions necessary for self-similar characteristics of the physical problem, at least until far downwind where the source height becomes negligible compared to the distance traveled and a second range of self-similarity is expected. Hence, the present self-similar model formulation will be a reasonable approximation only as long as boundary effects are not very significant, i.e., relatively close to the source.

The general solution of Eq. 15 can be shown to be

$$g(\hat{r}) \equiv g_*(\eta) = g_1(\eta)\int \frac{f_*(\eta)g_2(\eta)}{W(\eta)}d\eta - g_2(\eta)\int \frac{f_*(\eta)g_1(\eta)}{W(\eta)}d\eta \quad (17)$$

where $\eta = -(\hat{r}^2/2)$, $f_*(\eta) \equiv f(\hat{r})$, and

$$g_1(\eta) = {}_1F_1(\hat{a}, 1; \eta) \equiv \sum_{k=0}^{\infty} \frac{\Gamma(\hat{a} + k)}{\Gamma(\hat{a})\Gamma(1 + k)}\frac{\eta^k}{k!} \quad (18a)$$

$$g_2(\eta) = {}_1F_2(\hat{a}, 1; \eta) \equiv -\frac{1}{\Gamma(\hat{a})}\sum_{k=0}^{\infty} \frac{\Gamma(\hat{a} + k)}{\Gamma(\hat{a})\Gamma(1 + k)} \times \frac{\eta^k}{k!}[\psi(\hat{a} + k) - 2\psi(1 + k) + \ln\eta] \quad (18b)$$

$$\psi(x) \equiv \frac{\Gamma'(x)}{\Gamma(x)}$$

$$W(\eta) = W[{}_1F_1(\hat{a}, 1; \eta), {}_1F_2(\hat{a}, 1; \eta)] = -\frac{\exp(\eta)}{\eta\Gamma(\hat{a})} \quad (18c)$$

and

$$\hat{a} = \frac{(4 - \hat{\alpha})}{2}$$

${}_1F_1(\hat{a}, 1; \eta)$, ${}_1F_2(\hat{a}, 1; \eta)$ are confluent hypergeometric functions of the first and second kind, respectively (Abramowitz and Stegun, 1964; Lebedev, 1965) and are linearly independent. $W(\eta)$ is their Wronskian determinant and $\psi(x)$ is the logarithmic derivative of the gamma function. The constants of the integrations in Eq. 17 have to be calculated so as to satisfy the conditions of Eq. 16.

Alternatively, the boundary value problem defined by Eqs. 15–16 can be solved numerically for specific values of $\hat{\alpha}$. Csanady (1967, 1973) pursued this approach for the isotropic case assuming Gaussian $f(\hat{r})$, and presented typical $g(\hat{r})$ profiles together with the relative intensity of stream segregation $I_s \equiv \sigma_c^2/\langle c\rangle^2 = g\langle c_0\rangle^2/\langle c\rangle^2$. These calculations show that while the variance σ_c^2 [which is proportional to $g(\hat{r})$] decreases from the center of the plume to the fringes, by analogy to the mean concentration, the relative intensity of segregation—describing the degree of micromixing of the plume with the ambient—increases at the fringes. Near the plume centerline both quantities have very small gradients and thus can be considered approximately constant in a core region. For different values of $\hat{\alpha}$ different profiles of $g(\hat{r})$ are obtained. The center value $g(0)$ is a rapidly varying function of $\hat{\alpha}$. For $\hat{\alpha} > 4$ the origin becomes a saddle point (because $d^2g/d\hat{r}^2$ turns positive) and a full section across the plume will show a double-peaked profile for σ_c^2, a situation that is experimentally observed in free jets (Fischer et al., 1979; List, 1982). The physical reason is that the maximum rate of production occurs in the region of steepest gradients (around $\hat{r} = 1$), from where σ_c^2 diffuses both inward and outward. High diffusion and low dissipation (i.e., a low value of $\hat{\alpha}$) quickly smoothes the two peaks, resulting in a single peak at $\hat{r} = 0$. The problem in the development of self-similar solutions relies to a very large extent on the proper estimation of $\hat{\alpha}$. Csanady (1973) compared his approach to the experimental observations of Becker et al. (1966) by fitting $g(0)$ to the data. With the value of $g(0)$ obtained in this way, calculated profiles simulated measurements to a satisfactory degree with corresponding values of $\hat{\alpha}$ in the range 2.5 to 3.0. These values clearly are in very good agreement with the estimate $\hat{\alpha} = A_1/p$; indeed, for the theoretical values $p = 0.5$ and $A_1 = 1.5$, one obtains $\hat{\alpha} = 3.0$.

In conclusion, direct application of the self-similarity concept to the σ_c^2 transport problem, although it offers an integral representation of the solution of Eq. 11, does not lead to results appropriate for routine calculations (e.g., in conjunction with the common Gaussian solutions for $\langle c \rangle$). Indeed, the uncertainty in the parameters involved in Eq. 17 and the restricted range of conditions to which it applies would not justify the computational burden involved in its use. However, the conditions associated with the existence of self-similarity that are derived here are useful for reducing the complexity of the mathematical description of the fluctuations problem. These conditions will be used further in the next section combined with a scheme that is more appropriate for routine use than Eq. 15.

Localized Production of Fluctuations Model

In the following we present a new model that fulfills the need for simplicity by providing closed-form analytical expressions for $\sigma_c^2(x_1, y_2, y_3)$, using a limited number of parameters. This localized production of fluctuations (LPF) model is based on the knowledge of the nature of the terms of Eq. 11 and of its solutions, already discussed in the previous sections. Self-similarity of σ_c^2 profiles is not an *a priori* assumption in the development of this model; however, when assumed to hold, it simplifies further the structure of the final equations.

To avoid excessive notational complexity in this section we adopt a (x, y, z) coordinate system and drop the subscript R from the dispersion parameters; however it must be kept in mind that throughout the following discussion (x, y, z) are coordinates relative to the meandering plume centerline and K's, σ's, as well as $\langle c \rangle$ and σ_c^2, describe relative dispersion.

Model formulation

The solutions of Eq. 11 can in general be expressed in terms of the Green's function G of the corresponding nondissipative equation [containing only terms (i) and (iii)], through

$$\sigma_c^2(x, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^x G(x, y, z | x', y', z') \Pi_c(x', y', z') \times \exp\left[-\frac{1}{\bar{u}} \int_{x'}^{x} \frac{dx''}{t_d(x'')} \right] dx' dy' dz' \quad (19)$$

where $\Pi_c(x', y', z')$ is the spatial distribution of variance productio, given by Eq. 5.

Since production of σ_c^2 is of important magnitude, relevant to the other processes contributing to the balance of σ_c^2, mainly in the immediate vicinity of the source (where boundary effects can be neglected), an estimate of Π_c formulated in terms of a mean concentration field $\langle c \rangle$ for an unbounded flow should be a satisfactory approximation. For simplicity we consider the isotropic case; however, these results are directly extendable to the anisotropic case by an appropriate transformation of coordinates. In the isotropic case with $K_y(x) = K_z(x) = K(x)$, $\sigma_y(x) = \sigma_z(x) = \sigma(x) = \sigma_0 x^p$, one has

$$\Pi_c(x, r) = 2K(x) \left[\left(\frac{\partial \langle c \rangle}{\partial x} \right)^2 + \left(\frac{\partial \langle c \rangle}{\partial r} \right)^2 \right] = \Pi_x + \Pi_r \quad (20)$$

where $r = \sqrt{y^2 + z^2}$, $\Pi_x = 2K(x)(\partial \langle c \rangle / \partial x)^2$, $\Pi_r = 2K(x)(\partial \langle c \rangle / \partial r)^2$.

In general, studies of Eq. 11 have implicitly neglected production of σ_c^2 due to gradients of $\langle c \rangle$ in the x direction. In fact, locally [at a given point (x, r)] this term can be important; however, the total generation of fluctuations due to these gradients is small compared to the generation of gradients of $\langle c \rangle$ in the r direction. Indeed, for Gaussian mean instantaneous concentration distributions in the meandering frame of reference:

$$\langle c(x, r) \rangle = \langle c_0(x) \rangle \exp\left(-\frac{r^2}{2\sigma^2} \right) = \frac{S}{2\pi \bar{u} \sigma^2(x)} \exp\left(-\frac{r^2}{2\sigma^2} \right)$$

the ratio

$$\frac{\Pi_r}{\Pi_x} = \frac{2x^2 r^2}{p^2 (r^2 - 2\sigma^2)^2}$$

is not necessarily much larger than unity for arbitrary (x, r).

Consider, however, the overall crosswind fluctuations production at a given x from gradients of $\langle c \rangle$ in the r and x directions:

$$\Xi_r(x) = \int_0^{\infty} \int_0^{2\pi} r \Pi_r(x, r)\, d\phi dr = \frac{p}{x} \frac{S^2}{2\pi \bar{u} \sigma^2(x)} \quad (21)$$

$$\Xi_x(x) = \int_0^{\infty} \int_0^{2\pi} r \Pi_x(x, r)\, d\phi dr = \frac{2}{\exp 2} \sigma_0^2 p^3 x^{2p-3} \frac{S^2}{\pi \bar{u} \sigma^2(x)} \quad (22)$$

Statistical diffusion theory for small travel times demands that $p = O(1.0)$ for both the processes of absolute and relative diffusion. Thus, in the vicinity of the source $\Xi_r(x)/\Xi_x(x) = O(1/\sigma_0^2)$, which typically is much larger than unity and therefore the production of fluctuations due to gradients of $\langle c \rangle$ in the x direction can be neglected. Thus, finally, for the overall crossflow production of fluctuations $\Xi(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Pi(x, y, z)\, dydz$ one can write $\Xi(x) \simeq \Xi_r(x)$ with $\Xi_r(x)$ given by Eq. 21.

The formulation of the LPF model consists of two steps. The first step utilizes the fact that at every crossflow plane the production of fluctuations is strongly localized around its maximum value, which is attained at $r = \sigma$. Figure 2 shows the dimensionless distribution of radial production of fluctuations, $1/4x\Pi_r(x, r)(\bar{u})^{-1}p^{-1}\langle c_0(x) \rangle^{-2}$, with respect to r/σ, for arbitrary x. It is reasonable therefore to approximate the distribution of produc-

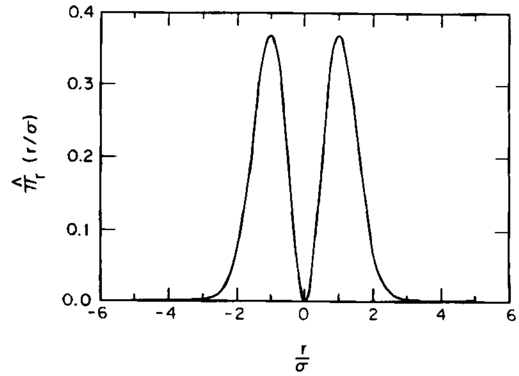

Figure 2. Dimensionless radial distribution of fluctuation production at any crossflow plane $\hat{\Pi}_r(r/\sigma) = (1/4)x\Pi_r(x, r/\sigma)(\bar{u})^{-1}p^{-1}\langle c_0(x) \rangle^{-2}$.

tion along a given radius by a delta function with strength estimated from Eq. 21. The optimal location of this delta function on each radius will be slightly off the value $r = \sigma$ since the production extends asymptotically to infinity; thus, for given ϕ, we fix this location at

$$r^* = \frac{\int_0^\infty r\Pi_r(x, r)rdr}{\int_0^\infty \Pi_r(x, r)rdr} = \Gamma\left(\frac{5}{2}\right)\sigma = \frac{3}{4}\sqrt{\pi}\sigma \tag{23}$$

i.e., at the center of mass of the actual production distribution. The complete locus of these delta functions at any x will be a ring of radius r^*.

Thus (for the isotropic case)

$$\Pi_c(x, r) = \Xi_r(x)\delta(r - r^*, \phi - \phi_s) \quad \text{with } r^* = \frac{3}{4}\sqrt{\pi}\sigma \tag{24}$$

where ϕ_s is arbitrary in the interval 0 to 2π.

This Π_c can be introduced in Eq. 19. However, (because of the nature of the Gaussian solution of $\langle c \rangle$) production of fluctuations is infinite at $x = 0$ and the integral would diverge. Of course, this is just an artifact created from the assumed ideal point character of the source producing the theoretical Gaussian solution that corresponds to a singularity at the origin. Since in reality the maximum field concentration is not infinite, it is justifiable to start the integration not at $x = 0$ but at some point $\hat{x}_0$. To apply Eq. 19, one should actually have to estimate $\hat{x}_0$ from available data so that it simulates measurements in a satisfactory manner. However, no general *a priori* estimate of it should be expected since it encompasses a variety of source and initial flow characteristics specific to each particular application. Another major problem that inhibits direct integration of Eq. 19 is the changing character of the relative dispersion process with downwind distance. Thus, fundamental two-particle dispersion theory predicts three asymptotic values for the exponent p, and even if one hypothesizes step changes and constant values in between, there is significant uncertainty regarding the location of these changes; similar uncertainties are associated with the σ_0's. We circumvent these problems by introducing the second step in the formulation of the LPF model. Applying the mean-value theorem of Lagrange to the isotropic form of Eq. 19 for the integration with respect to x, one has

$$\sigma_c^2(x, y, z) = \hat{\Xi}(\xi, x)\int_0^\infty\int_0^{2\pi}\frac{1}{r'}\delta(r' - r^*) \times \delta(\phi' - \phi_s)G(x, r, \phi|\xi, r', \phi')r'd\phi'dr' \times \exp\left[-\frac{1}{\bar{u}}\int_\xi^x \frac{dx_1}{t_d(x_1)}\right] \tag{25}$$

with

$$\hat{\Xi}(\xi, x) = \frac{S^2}{4\pi\bar{u}\sigma_0^2}2p\xi^{-2p-1}(x - \hat{x}_0) \tag{26a}$$

where ξ is some point between $\hat{x}_0$ and x (fixed for given $\hat{x}_0$, x). Setting $\xi = \omega x$, with $0 < \omega \leq 1$, and assuming that $x \gg \hat{x}_0$ one can further write

$$\hat{\Xi}(x) = \frac{pS^2}{2\pi\bar{u}\sigma_0^2\xi^{2p}}\frac{x}{\xi} = \frac{\omega pS^2}{2\pi\bar{u}\sigma^2(\omega x)} \tag{26b}$$

So, the problem of estimating $\hat{x}_0$, or, more generally, integrating Eq. 19, is essentially transferred to the problem of choosing the appropriate value (between 0 and 1) of the dimensionless localization parameter ω (that can possibly vary, within these limits, with distance x). Now, Eq. 25 associates σ_c^2 at x to the dispersion parameters corresponding only to x and to another single ωx. In this way all the different kinds of uncertainty implicit in Eq. 19 are now collectively lumped in one parameter, i.e., in the unknown value of ω.

Analytical solutions

Equation 19 can now be used, through its reduced form, Eq. 25, to obtain approximate closed solutions to the variance transport Eq. 11. For an unbounded flow (and $\sigma_c^2 \rightarrow 0$ at $y, z \rightarrow \infty$) the corresponding Green's function of Eq. 11 [without terms (iv) and (ii)], is

$$G(x, y, z|x', y', z') = G(x - x', y - y', z - z') = \frac{1}{2\pi\sigma_y(x - x')\sigma_z(x - x')\bar{u}} \times \exp\left[-\frac{(y - y')^2}{2\sigma_y^2(x - x')} - \frac{(z - z')^2}{2\sigma_z^2(x - x')}\right] \tag{27}$$

when dispersion is assumed negligible compared to advection in the x direction and the σ's are related to the K's through Eq. 4.

Using Eq. 9 one obtains

$$\int_{x'}^x \frac{dx''}{t_d(x'')} = A_1\bar{u}[\ln(x + x_0) - \ln(x' + x_0)]$$

and

$$\exp\left[-\frac{1}{\bar{u}}\int_{x'}^x \frac{dx''}{t_d(x'')}\right] = \left(\frac{x' + x_0}{x + x_0}\right)^{A_1} \tag{28}$$

Consider the general anisotropic (orthotropic) case, where the source of fluctuations takes the form of an elliptical ring (of infinitesimal thickness) located at $x = \xi$ with semiaxes a, b such that $a = 3/4\sqrt{\pi}\sigma_y(\xi)$, $b = \kappa a$, where $\kappa = \sigma_z(\xi)/\sigma_y(\xi)$. We define $w^2 = ab$ and $\sigma^2(x) = \sigma_y(x)\sigma_z(x)$ and introduce the new variables $y_1 = \kappa^{1/2}y$ and $z_1 = \kappa^{-1/2}z$. The transformation $(y, z) \mapsto (y_1, z_1)$ has a Jacobian equal to unity and therefore preserves areas. The phenomenal variance source coordinates will transform to $y_{1s} = w\cos\phi_s$, $z_{1s} = w\sin\phi_s$ where ϕ_s is now the polar angle of point (y_{1s}, z_{1s}) in the new coordinate system. The Cartesian form of Eq. 25 in this system will be

$$\sigma_c^2 = \frac{\hat{S}(\xi)}{2\pi\bar{u}\sigma^2(x - \xi)}\left(\frac{\xi + x_0}{x + x_0}\right)^{A_1} \times \int_{-\infty}^\infty\int_{-\infty}^\infty \exp\left[-\frac{(y_1 - y_1')^2 + (z_1 - z_1')^2}{2\sigma^2(x - \xi)}\right] \delta(y_1' - y_{1s})\delta(z_1' - z_{1s})dy_1'dz_1'$$

where $\hat{S}(\xi) = (pS^2)/[2\omega\pi\bar{u}\sigma_y(\xi)\sigma_z(\xi)]$.

Introducing polar coordinates $r_1 = \sqrt{y_1^2 + z_1^2}$, $y_1 = r_1 \cos\phi$, $z_1 = r_1 \sin\phi$, the integral in the above relation becomes

$$\int_0^\infty \int_0^{2\pi} \frac{1}{r_1'} \delta(r_1' - w)\delta(\phi' - \phi_s) \exp\left[-\frac{R^2}{2\sigma^2(x-\xi)}\right] r_1' d\phi' dr_1'$$

where $R^2 = (y_1 - y_1')^2 + (z_1 - z_1')^2 = r_1^2 + r_1'^2 - 2r_1r_1' \cos(\phi - \phi')$. Thus,

$$\sigma_c^2 = \frac{\hat{S}(\xi)}{2\pi\bar{u}\sigma^2(x-\xi)}\left(\frac{\xi + x_0}{x + x_0}\right)^{A_1} \exp\left[-\frac{r_1^2 + w^2}{2\sigma^2(x-\xi)}\right] \times \int_0^{2\pi} \exp\left[\frac{r_1 w \cos(\phi' - \phi_s)}{2\sigma^2(x-\xi)}\right] d\phi'$$

which finally gives

$$\sigma_c^2 = \frac{\hat{S}(\xi)}{2\pi\bar{u}\sigma^2(x-\xi)}\left(\frac{\xi + x_0}{x + x_0}\right)^{A_1} \times \exp\left[-\frac{r_1^2 + w^2}{2\sigma^2(x-\xi)}\right] I_0\left[\frac{r_1 w}{\sigma^2(x-\xi)}\right] \quad (29)$$

where $I_0(\cdot)$ is the modified Bessel function of order zero.

For $x, \xi \gg x_0$ Eq. 29 becomes

$$\sigma_c^2(x, r_1; \xi) = \left(\frac{S}{2\pi\bar{u}}\right)^2 \frac{p\omega(x)^{A_1 - 1}}{\sigma^2(\xi)\sigma^2(x-\xi)} \times \exp\left[-\frac{r_1^2 + w^2}{2\sigma^2(x-\xi)}\right] I_0\left[\frac{r_1 w}{\sigma^2(x-\xi)}\right] \quad (30)$$

Now, a sufficient condition for self-similarity of the σ_c^2 profiles for a given range of x (where p, A_1 are assumed constant) is that ω is a constant in this range. In this case $\sigma(\xi) = \omega^p\sigma(x)$, $\sigma(x - \xi) = (1 - \omega)^p\sigma(x)$, $w = 3/4\sqrt{\pi}\omega^p\sigma(x)$, and σ_c^2 becomes

$$\sigma_c^2 = \langle c_0(x)\rangle^2 g\left[\frac{r_1}{\sigma(x)}\right] = \langle c_0(x)\rangle^2 g(0) \times \exp\left(\frac{-1}{2\bar{\omega}^{2p}} \frac{r_1^2}{\sigma^2}\right) I_0\left(\frac{3\sqrt{\pi}\omega^p}{4\bar{\omega}^{2p}} \frac{r_1}{\sigma}\right) \quad (31)$$

where $\bar{\omega} = 1 - \omega$ and $g(\cdot) = I_s\langle c\rangle^2/\langle c_0\rangle^2$ is the dimensionless absolute intensity of internal fluctuations. On the plume centerline $g(0)$ will be

$$g(0) = \frac{p\omega^{A_1 - 2p - 1}}{(\bar{\omega})^{2p}} \exp\left(-\frac{9\pi\omega^{2p}}{32\bar{\omega}^{2p}}\right) \quad (32)$$

When experimental information for this quantity is available it can be used in conjunction with information on parameters p and A_1 to estimate ω values (see the next section).

Returning to the y, z coordinates, Eq. 30 takes the form

$$\sigma_c^2(x, y, z; \xi) = \left(\frac{S}{2\pi\bar{u}}\right)^2 \frac{p\omega(x)^{A_1 - 1}}{\sigma_y(\xi)\sigma_z(\xi)\sigma_y(x-\xi)\sigma_z(x-\xi)} \times \exp\left[-\frac{y^2}{2\sigma_y^2(x-\xi)}\right] \exp\left[-\frac{z^2}{2\sigma_z^2(x-\xi)}\right] \times \exp\left[-\frac{ab}{2\sigma_y(x-\xi)\sigma_z(x-\xi)}\right] I_0\left[\frac{\sqrt{a^2z^2 + b^2y^2}}{\sigma_y(x-\xi)\sigma_z(x-\xi)}\right] \quad (33)$$

Equations 31 and 33 constitute basic, usable, forms of the LPF model when boundary effects can be assumed to be negligible, as in the immediate vicinity of the source.

When the dispersion field cannot be assumed unbounded, one must take into account the boundary condition of Eq. 10, i.e., $\sigma_c^2 \rightarrow 0$ at $z(= y_3) = -b_3$, where, because of meandering effects b_3 is a random variable. Far downstream, where meandering is negligible and boundary effects most significant, one can obtain the following result, assuming that the plume centerline is at a constant height h from the boundary (notice that now the coordinates' origin is fixed on the boundary):

$$\sigma_c^2(x, y, z; \xi) = \left(\frac{S}{2\pi\bar{u}}\right)^2 \frac{p\omega(x)^{A_1 - 1}}{\sigma_y(\xi)\sigma_z(\xi)\sigma_y(x-\xi)\sigma_z(x-\xi)} \times \exp\left[-\frac{y_2}{2\sigma_y^2(x-\xi)}\right] \exp\left[-\frac{ab}{2\sigma_y(x-\xi)\sigma_z(x-\xi)}\right] \times \left\{\exp\left[-\frac{(z-h)^2}{2\sigma_z^2(x-\xi)}\right] I_0\left[\frac{\sqrt{a^2(z-h)^2 + b^2y^2}}{\sigma_y(x-\xi)\sigma_z(x-\xi)}\right] - \alpha \exp\left[-\frac{(z+h)^2}{2\sigma_z^2(x-\xi)}\right] I_0\left[\frac{\sqrt{a^2(z+h)^2 + b^2y^2}}{\sigma_y(x-\xi)\sigma_z(x-\xi)}\right]\right\}$$

The parameter α appearing in the above equation equals unity for a perfectly absorbing boundary, i.e., σ_c^2 actually equal to 0 at the surface. However, the effects of dissipation might not be so strong, and a lower value for α may be more appropriate.

Model Testing and Discussion

The LPF model is a simple formulation for the internal concentration variance that is directly derived from the physics of the point release problem, starting from rigorous equations and utilizing empirical information and approximations to simplify the analysis. The required inputs reduce to a set of physical parameters and a model-specific parameter. The physical parameters are the relative dispersion σ's (which are assumed to obey simple power laws, at least locally) and a factor relating dispersion time to the local dissipation time scale t_d. The model-specific or localization parameter ω actually defines the location of an effective source of fluctuations. As already mentioned, uncertainties associated with a variety of factors such as source size, flow conditions, and the relative dispersion process itself, are lumped into ω. Introduction of ω reduces the uncertainty associated with the physical parameters since, instead of their complete—and unknown—variation with downstream distance, only estimates of their local values are needed.

The problem of estimating ω is facilitated by two facts:

1. The often-observed validity of self-similarity, which is expressed by Eq. 12
2. The observation that $g(0)$ is at most a weak function of the distance from the source, even for the total fluctuations variance (Sawford et al., 1985). This $g(0)$ for given flow conditions tends to a constant value after a certain distance (Becker et al., 1966; Fackrell and Robins, 1982ab; Wilson et al., 1982ab, etc).

These facts, although deduced from observations that do not cover the entire range of possible conditions encountered in laboratory and environmental flows, suggest that adequately reliable estimates of ω are possible, at least for specified ranges of the dispersion, even without a complete understanding and analysis of all the mechanisms that affect the level of $g(0)$. It is therefore often justified to treat $g(0)$ as an empirical constant typical of given flow conditions. In this simplified approach ω is completely determined from the physical parameters [including $g(0)$] of the problem. Of course in order to be able to construct empirical estimates of, say, typical values of $g(0)$ (and therefore ω) for ambient turbulence of various Reynolds numbers, many more experimental data bases than are currently available are needed. In a more fundamental approach $g(0)$ or closely related functions have been modeled theoretically, in terms of statistical correlations of the turbulent flow field, for source configurations that create mean concentration fields approximately equivalent to that of the continuous point source. Numerical simulations and analytical expressions that in general involve a measure of effective source size are available (Durbin, 1980, 1982; Sawford, 1983). However, because of the existing uncertainties and limitations in the formulation of the theoretical models, it presently seems reasonable to confine this discussion to the previously mentioned simplified approach.

As far as atmospheric dispersion is concerned, present knowledge suggests that for neutral stability the far field value of $g(0)$ is of order unity (for elevated sources), and use of the typical (theoretical) values $p = 0.5$ (for the far field), $A_1 = 1.5$ to 2.5 seems to offer a qualitatively acceptable simulation of many available relevant field and wind tunnel data sets. (For example, see Figure 5). However the scatter, the resolution, and the uncertainty of these data often make quantitative comparisons meaningless or impossible. The problems are even more severe in cases of more complicated atmospheric conditions.

The measurements most appropriate for comparing with and testing LPF model calculations are those of Becker et al. (1966) for point-source dispersion in homogenous, quasi-isotropic, pipe flow turbulence. Indeed, in the conditions of these experiments meandering was insignificant and the structure of the turbulent flow, being relevant to the conditions for which Eq. 8 was suggested, reduces the uncertainty regarding the proper choice of A_1; further, $p = 0.5$ fits accurately the entire range of the data. Thus, the uncertainty regarding the physical parameters is minimum. Self-similarity of σ_c^2 profiles and a constant value of $g(0)$ are observed in all these experiments. Comparisons of LPF calculations with reported absolute and relative intensities of internal fluctuations are shown in Figures 3a, 3b, 3c, and 4. The

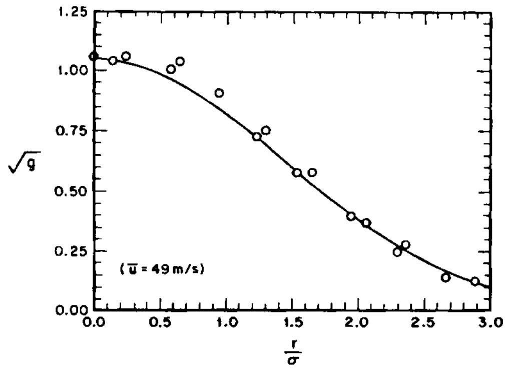

Figure 3b. LPF model prediction of $\sqrt{g(r/\sigma)}$ (solid line) compared with data from Becker et al. (1966) (circles).

Data at two downstream distances, centerline velocity 49 m/s. LPF model calculations for $p = 0.5, A_1 = 1.5$ [$g(0) = 1.05$].

parameter ω is estimated directly (for $A_1 = 1.5$) from the centerline value $g(0)$, whose square root value, for all the flows studied, lies in the range 1.0 ± 0.2. The agreement obtained by using solely the centerline value to adjust ω, while p and A_1 are preset equal to their theoretical values, must be considered very satisfactory. (Somewhat different values of A_1 can improve slightly the success of the simulation, especially near $r/\sigma = 0.75$ where the difference between predictions and observations seems higher.)

A comparison with atmospheric field data is also shown in Figure 5. The data are of Ramsdell and Hinds (1971) and the typical values $p = 0.5, A_1 = 1.5$ were used, while ω is determined directly by the centerline intensity. Although the uncertainty of the data is very significant the agreement can be considered satisfactory in this case too.

Conclusions

Knowledge of statistical properties of point-source plume concentrations, such as the variance σ_c^2 or the intensity of segregation, is essential in many situations calling for plume modeling (e.g., in assessing the impact of releases of pollutants in the environment), and in particular in estimating the effects of local tur-

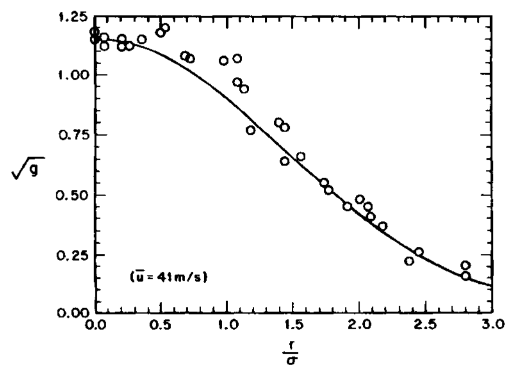

Figure 3a. LPF model prediction of dimensionless absolute square root intensity of segregation $\sqrt{g(r/\sigma)}$ (solid line) compared with data from Becker et al. (1966), (circles).

Data at five downstream distances, centerline velocity 41 m/s. LPF model calculations for $p = 0.5, A_1 = 1.5$ [$g(0) = 1.15$].

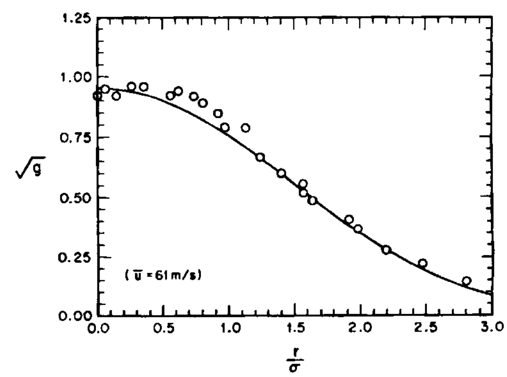

Figure 3c. LPF model prediction of $\sqrt{g(r/\sigma)}$ (solid line) compared with data from Becker et al. (1966) (circles).

Data at four downstream distances, center line velocity 61 m/s. LPF model calculations for $p = 0.5, A_1 = 1.5$ [$g(0) = 0.95$].

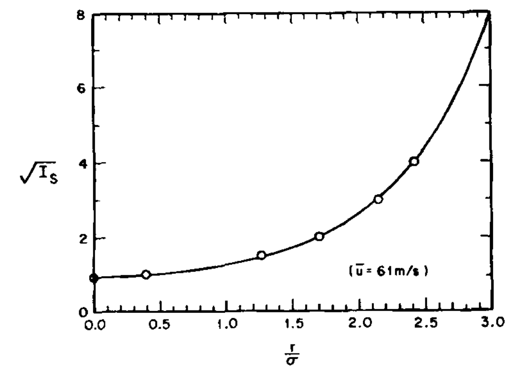

Figure 4. LPF model prediction of dimensionless relative square root intensity of segregation $\sqrt{I_s}$ (solid line) compared with data from Becker et al (1966) (circles).

Data at five downstream distances, centerline velocity 61 m/s.
O O O correspond to experimental curve in Fig. 7 of Becker et al., in which $\sqrt{I_s}$ is plotted vs. $r/r_{1/2}$.
LPF model calculations for $p = 0.5$, $A_1 = 1.5$.

bulent mixing on relatively fast nonlinear chemistry. Although recent experimental and theoretical work has enhanced significantly the available information on the behavior of σ_c^2, this had not resulted in the development of practical predictive methods, especially with regard to fine-scale in-plume fluctuations—as opposed to total observed fluctuations that encompass bulk motion effects (meandering), which do not interact with the chemical processes.

A new model for the "internal" σ_c^2, at a level of sophistication analogous to that of the Gaussian formulas for the mean concentration field, has been developed here, starting from the Eulerian transport equation for σ_c^2. A series of approximations utilizing existing experimental and theoretical information for the processes involved, combined with the localized production of fluctuations (LPF) scheme allowed the construction of closed analytic expressions for σ_c^2, directly from its governing equation. The capability of this LPF model to simulate the variance profile was successfully tested against available data on point source plume concentrations.

In conclusion, the model developed in this work provides a rational, yet computationally simple, means for describing concentration fluctuations and the corresponding intensity of segregation inside instantaneous plume boundaries. Although its applicability is restricted by assumptions such as the uniform mean flow (or "mild" mean plume motion), and slender plumes, this model can serve as a first approximation to a wide range of point-source dispersion problems. Further experimental information will be useful to provide accurate estimates of its parameters for specific flows.

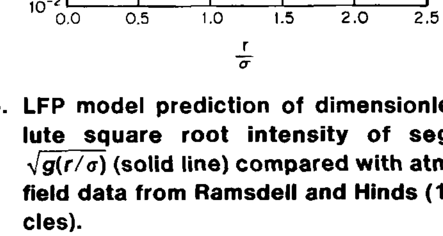

Figure 5. LFP model prediction of dimensionless absolute square root intensity of segregation $\sqrt{g(r/\sigma)}$ (solid line) compared with atmospheric field data from Ramsdell and Hinds (1971) (circles).

LPF model calculations for $p = 0.5$, $A_1 = 1.5$.

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Notation

A_1 = reciprocal decay time scale factor, Eq. 9
$\hat{a}$ = similarity solution parameter, Eq. 18
a = horizontal semiaxis of the phenomenal variance source
b = vertical semiaxis of the phenomenal variance source
b_i = instantaneous plume centerline coordinates
c = instantaneous concentration
c_R = instantaneous concentration notation used in Fig. 1 for values in the meandering frame of reference
D = molecular diffusivity
${}_1F_1, {}_1F_2$ = confluent hypergeometric functions of the 1st and 2nd kind, respectively
f = similarity function for $\langle c \rangle$
$G(\cdot|\cdot)$ = Green's function
g = local absolute intensity of segregation
h = distance of mean plume centerline from boundary
$I_0(\cdot)$ = modified Bessel function of order zero
I_s = local relative intensity of segregation
K_i = eddy diffusivity in the i direction
K_{Ri} = eddy diffusivity for relative dispersion in the i direction
k = dissipation time scale factor, Eq. 8
k_0 = constant in dissipation approximation, Eq. 8
ℓ_d = dissipation length scale, Eq. 7
p = exponent in dispersion law
r = radial distance (isotropic conditions)
$\check{r}$ = weighted squared radial distance (anisotropic conditions)
$\hat{r}$ = dimensionless (weighted or nonweighted) radial distance
r^* = radius of the phenomenal source of variance production (isotropic conditions), Eq. 23
$\check{s}$ = $\sigma_{R2}\sigma_{R3}$
t = dispersion time
t_d = dissipation time scale, Eq. 7
t_0 = constant in dissipation approximation, Eq. 9
u_i = random velocity component in the i direction
$\bar{u}$ = uniform mean ambient velocity
x_i = fixed frame coordinates
x_0 = constant in dissipation approximation, Eq. 9
$\hat{x}_0$ = empirical lower integration limit for Eq. 19
y_i = meandering frame coordinates
x, y, z = meandering frame coordinates

Greek letters

α = surface absorption factor for σ_c^2
$\hat{\alpha}$ = similarity condition parameter, Eq. 14
$\Gamma(\cdot)$ = gamma function
η = $-\hat{r}^2/2$
κ = similarity condition parameter, Eq. 13
Ξ_x = total production of fluctuations for given x due to gradients in the x direction
Ξ_r = total production of fluctuations for given x due to gradients in the r direction
ξ = ωx
Π_c = local production of fluctuations at a point, Eq. 5

Π_x = local production of fluctuations at a given point due to gradients in the x direction

Π_r = local production of fluctuations at a given point due to gradients in the r direction

σ_c^2 = variance of the instantaneous concentration field of a point source plume in a meandering frame

σ_{Ri}, σ_i = relative dispersion parameter in the i direction

Φ = local dissipation of fluctuations, Eq. 7

$\psi(\cdot)$ = logarithmic derivative of the gamma function

ω = dimensionless localization parameter ($0 < \omega < 1$)

$\overline{\omega} = 1 - \omega$

Special symbols

$\langle \cdot \rangle$ = ensemble mean of random variable

$(\cdot)'$ = fluctuation of the random variable about its ensemble mean

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